Dual Consensus Measure for Multi-Perspective Multi-Criteria Group Decision Making

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***Abstract*—This paper investigates the problem of measuring consensus in multi-perspective Multi-Criteria Group Decision Making (MCGDM) problems, in which participants have individual views on the relative importance of different evaluation criteria. A novel dual consensus measure for multi-perspective MCGDM problems is introduced. The proposed measure determines the level of agreement between participants' opinions based on: (i) the global performance or satisfaction of alternatives, (ii) their partial performances of alternatives under each criterion, and (iii) the similarity between the perspectives of participants regarding criteria weights. Preliminary experiments are conducted for an example multi-perspective MCGDM scenario. The degree to which global and partial performance information are jointly taken into account - together with the actual pairwise distances between the opinions of participants - are shown to directly affect the overall measurement of consensus in the group. An application example is introduced in a MCGDM problem on selecting the safest logistic route to transport hazardous materials.**

I. INTRODUCTION

Group Decision Making (GDM) problems are decision situations in which multiple participants, or *experts*, with distinct background and interests attempt to make a collective decision on ranking of a finite set of alternatives or to the selection of the best or most suitable one(s). When participants must evaluate the satisfaction or performance of each alternative according to multiple criteria separately, we have a Multi-Criteria GDM (MCGDM) problem. Conventional GDM and MCGDM approaches involve a selection process in which the experts' preferences are combined into an aggregated collective preference utilized to determine the solution for the decision problem. Notwithstanding, many real-life collective decisions demand alleviating internal disagreements and achieving a high level of agreement among participants [1]. Hence, consensus reaching processes are introduced in an attempt to bring the initial preferences of experts closer to each other before making the group decision. A large body of research [6] has been devoted in recent decades to defining consensus-driven (MC)GDM models, consensus support systems, and consensus measures to determine the level of group agreement based on the individual preferences of participants.

Consensus measures have been scarcely investigated in the existing MCGDM literature, with few works focused on consensus approaches for multiple-criteria decision contexts (compared to consensus-based GDM approaches [6]). Xu et al. [10] pointed out the necessity of consensus reaching approaches in MCGDM, and proposed a consensus model for MCGDM problems under an uncertain linguistic setting. Although their consensus model and underlying consensus measure takes account of decision weight information related to the importance of participants, it is assumed that all criteria are equally important. In [7], Pang et al. introduced a consensus framework for MCGDM with uncertain linguistic assessments. The underlying consensus measure relies on determining, for each alternative x_j and pair of experts $e_i, e_{i'}$, two closeness degrees: (i) pairwise closeness for x_j among the upper limits of the uncertain linguistic assessments, and (ii) pairwise closeness for x_j among the lower limits of such assessments. Both upper and lower pairwise similarities are then fused into a single pairwise consensus degree on the alternative. Yan et al. in [12] defined a consensus measure between individual and collective preferential information in linguistic MCGDM. Their measure is based on the concept of stochastic dominance, a probabilistic interpretation of importance weights assigned to the participants, and the concept of fuzzy majority. Concretely, the authors define a new consensus measure along with its main properties, predicated on the deviation degree between two random preferences generated by a probability distribution resulting from aggregating the original experts' assessments. A consensus measure for MCGDM with linguistic preference relations was established by Sun and Ma in [9]. Their proposed method not only measures the level of agreement based on linguistic interval comparison operations, but also allows for flexibility in calculating an acceptable threshold agreement level. However, although their method is envisaged for a MCGDM framework, it requires eliciting separate preference relations in the cases when several criteria must be considered. In [3], Choudhury et al. presented an intelligent consensus-driven decision model for advanced technology selection. Three dimensions of criteria, namely strategic, technological and social factors, are considered in the target scenario, along with heterogeneous preference structures and formats (preference orderings, utility functions, fuzzy preference relations, multiplicative preference relations). Notably, Choudhury et al.'s preference information does not involve assessments and pairwise comparisons across various alternatives , but rather direct assessments of the relative importance of criteria.

In some consensus measures defined in the above studies [7], [12], the criteria weights are taken into consideration in the underlying aggregation processes to obtain the collective consensus degrees. Other approaches [3] do not accommodate assessing multiple alternatives under several criteria within

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the same preference structure. Moreover, not all existing consensus measures for MCGDM consider different importance weights for criteria in their underlying computations [9], [10], [12], and hence, such criteria weights are taken into account during the subsequent aggregation and alternative(s) exploitation processes exclusively. Nevertheless, the interest in this study lies in the fact that existing consensus approaches for MCGDM in the literature [7], [9], [10], [12] assume that the evaluation criteria have associated a unique set of importance weights for the whole decision problem at hand. In other words, the weights assigned to criteria are common to the entire decision group. By contrast, in this paper we investigate multi-perspective MCGDM problems, i.e. problems in which each expert has his/her own perspective on the relative importance weights of the evaluation criteria. We formulate the following assumptions: (i) the consensus measures defined in existing works for MCGDM are not suitable to accommodate multi-perspective MCGDM situations in which both the individuals' opinions and their perspectives on the available criteria must be taken into consideration throughout the process of measuring and building consensus, and (ii) the effect of using a consensus measure varies depending on whether it considers the level of agreement/disagreement among participants on the global performance of each alternative, or separately for the alternative performance under each criterion.

To illustrate the second assumption above, consider the following motivation example (whose employed notations are formally introduced later in Section II-A). A family with four members wishes to choose a new car to buy from several available car models. Each of the family members subjectively assesses each candidate car model in accordance with three criteria: c_1: comfort, c_2 : design and c_3: equipment. Assume that the first family member, e_1, provides the following assessments in the unit interval [0,1] for model x_1: $p_1^1 = (0, 0.75, 1)$. Furthermore, for e_1 all three criteria are moderately important, with the second one (design) being slightly less important than the other two, thus e_1's importance weights for criteria are $W_1 = [0.4 \quad 0.2 \quad 0.4]^T$. Similarly, e_2 supplies the following assessments for x_1 and importance weights for the existing criteria, $p_2^1 = (0.75, 1, 0)$, $W_2 = [0.2 \quad 0.4 \quad 0.4]^T$. For e_1, an overall assessment of x_1, denoted $\mathcal{P}_1(x_1)$ and indicating the satisfaction degree (or performance, as referred to herein) of x_1, can be calculated as the criteria-weighted average of her three assessments on x_1, which yields $\mathcal{P}_1(x_1) = 0.55$. For e_2, we also have $\mathcal{P}_2(x_1) = 0.55$. Therefore, measuring consensus between e_1 and e_2 based on their global performance on x_1 would indicate that they fully (unanimously) agree on their opinion for x_1. However, a consensus measure that looked at differences between assessments for each criterion separately, would easily identify that e_1 is completely satisfied with the equipment criterion of x_1, whereas e_2 is completely dissatisfied with x_1's equipment. This situation clearly contradicts the full agreement among their global assessments on x_1. Moreover, both members have different perspectives on the relative importance of criteria (e.g. for e_1, comfort is more crucial than design, and vice versa for e_2), and this situation, such information is lost in the aggregation process.

The above example shows that measuring consensus on the *global performance* of an alternative yields different results than those obtained by reflecting the agreement positions on assessments separately for each criterion, i.e. *partial performances* of the alternative. Although both approaches follow a similar rationale to existing consensus approaches in MCGDM scenarios, using one or another in isolation might lead to unreliable results, in which relevant decision information has not been taken into account properly. The objective in this work is to investigate and define a "compromise" dual consensus measure for multi-perspective MCGDM problems, that meaningfully incorporates both "views" jointly. To do this, related preliminaries on MCGDM and consensus measures are firstly overviewed in Section 2. In Section 3, two distance metrics among participants' opinions are introduced: (i) distance based on the the global performance of each alternative "as a whole"; and (ii) distance based on the partial performances of such an alternative for each criterion. Both distances are subsequently combined to define a novel dual consensus measure (illustrated in Section 3.4 for the selection of the safest logistic route to transport hazardous materials) that comprehensively reflects the agreement level based on both global and partial performances of each alternative. The proposed consensus measure also takes the similarity between individual perspectives on criteria into account. Section 4 concludes the paper.

II. PRELIMINARIES

A. Decision-Making Framework

A MCGDM problem is defined as a situation in which a set of at least two experts, $E = \{e_1, e_2, \ldots, e_m\}$ $(m \geq 2)$ attempt to jointly select the best alternative(s) out of a finite set, $X = \{x_1, x_2, \ldots, x_n\}$, $(n \geq 2)$, or to rank them in decreasing order by satisfiability. Given a finite set of evaluation criteria, $C = \{c_1, c_2 \ldots, c_l\}$, $(l \geq 1)$, each expert e_i supplies an assessment matrix, $P_i = (p_i^{jk})_{n \times l}$, in which each assessment p_i^{jk}, expressed in an assessment format D, indicates e_i's opinion on the performance or satisfiability degree of x_j under the kth criterion. Examples of assessment formats frequently considered in MCGDM literature include numerical values in the unit interval, interval-valued, triangular fuzzy numbers, linguistic terms, etc [11], [13]. Within the scope of this paper, we assume the use of quantitative benefit[1] assessments lying in the unit interval. We also consider a particular case of MCGDM problems referred to as *multi-perspective* MCGDM, in which each group member $e_i \in E$ has their own view of the relative importance of criteria, represented as an individual weighting vector $W_i = [w_i^1 \quad \ldots \quad w_i^l]^T$, such that $w_i^k \in [0, 1]$ and $\sum_{k=1}^{l} w_i^k = 1$.

B. Consensus measures in Group Decision Making

In most practical GDM and MCGDM problems, obtaining a solution deemed as acceptable by the whole group becomes

[1]Several MCGDM approaches distinguish between *cost* and *benefit* criteria, such that assessments on both types of criteria are interpreted differently. We assume hereinafter that higher assessment values imply a better performance of the alternative under each criterion.

paramount. Consensus reaching processes are introduced in such cases to foster the interaction among participants and to modify their preferences, so as to bring them closer to each other and increase the level of collective agreement [6]. There are various interpretations to the concept of consensus, the most strict of which views consensus as full agreement or *unanimity*. However, achieving consensus as unanimity is in practice infeasible in most real-life decision making contexts. As a counterpart to the strict view of consensus as unanimous agreement, flexible interpretations implying that consensus can be measured under different levels of partial collective agreement, have been widely adopted in numerous consensus-based (MC)GDM approaches [2], [5]. Based on individual preferential information provided by participants in a group, a *consensus measure* [8] determines the level of agreement in a group or *consensus degree*, indicating how close (or distant) the individual opinions are from unanimity. This consensus degree is numerical value in the unit interval (such that the closer the value is to 1, the closer the group is to unanimity), which is compared against a threshold consensus degree or minimum level of consensus required, established a priori.

There exist two broad types of consensus measures, as categorized in Palomares et al.'s survey [6].

- *Consensus measures based on distances to the collective preference*: These measures firstly determine the collective preference P_c representing the overall opinion of the group. Each assessment in the collective preference is obtained by aggregating m individual assessments of participating experts, using an aggregation function ϕ, i.e. $p_c^{jk} = \phi(p_1^{jk}, p_2^{jk} \ldots, p_m^{jk})$. Consensus degrees are then obtained at assessment level based on the distance between each individual opinion and the collective opinion, and finally aggregated into a global consensus degree. Lower distance values between each individual opinion and the collective opinion contribute to a higher level of consensus, and vice versa.
- *Consensus measures based on pairwise distances between experts*: For each pair of experts $e_i, e_{i'}$, $i \neq i'$, the distance between their opinions (or conversely, the similarity between them, as considered in several consensus models) is computed at assessment level. Pairwise distances (resp. similarities) are then aggregated into assessment-level consensus degrees for the group. Successive aggregation of such degrees finally yields the global consensus degree.

III. Dual Consensus Measure for Multi-Perspective Multi-Criteria Group Decision Making

This section introduces a novel dual consensus measure for multi-perspective MCGDM problems, which combines two pairwise distance measures between experts' opinions. The proposed consensus measure jointly captures the level of agreement between experts' preferential information based on (i) the global performance or satisfaction of alternatives, (ii) the partial performances of alternatives under each criterion, and (iii) the similarity between experts' perspectives on the relative importance of criteria. Sections III-A and III-B introduce the global and partial distance metrics underlying the proposed dual consensus measure, which is formalized in Section III-C. For the sake of simplicity and illustration, the examples introduced in the sequel assume a discrete, quantitative assessment scale across the unit interval (see Table I).

Assessment value	Semantics
0.0	Absolutely unsatisfactory
0.25	Unsatisfactory
0.5	Neither unsatisfactory nor satisfactory
0.75	Satisfactory
1.0	Absolutely satisfactory

TABLE I: Assessment scale utilized to illustrate the proposed dual consensus measure

A. Distance based on Global Alternative Performance

Let $P_i, P_{i'}$ denote the multi-criteria evaluation matrices provided by the ith and i'th experts, $e_i \in E$ on X, $i \in \{1, \ldots, m\}$. Let $p_i^j = (p_i^{j1}, \ldots, p_i^{jl})$ be e_i's preference vector containing her/his assessments on the jth alternative. Intuitively, p_i^j coincides with the jth row in the evaluation matrix P_i. Similarly, denote by $p_{i'}^j = (p_{i'}^{j1}, \ldots, p_{i'}^{jl})$ the assessments elicited from $e_{i'}$ on x_j. Let $W_i = [w_i^1 \ldots w_i^l]^T$ and $W_{i'} = [w_{i'}^1 \ldots w_{i'}^l]^T$ be their individual weighting vectors of decision criteria. Based on p_i^j, W_i, $p_{i'}^j$ and $W_{i'}$ we define the distance between the ith expert's opinion and the i'th expert's opinion regarding the global performance of x_j, denoted by $d_G(p_i^j, p_{i'}^j)$, as follows:

$$d_G(p_i^j, p_{i'}^j) = \left| \sum_{k=1}^{l} w_i^k \cdot p_i^{jk} - \sum_{k=1}^{l} w_{i'}^k \cdot p_{i'}^{jk} \right| \tag{1}$$

where $\sum_{k=1}^l w_i^k \cdot p_i^{jk}$ and $\sum_{k=1}^l w_{i'}^k \cdot p_{i'}^{jk} = \mathcal{P}(x_i)$ represent the global performance of x_j based on e_i's individual assessments and the (aggregated) collective assessments, respectively. It can be easily verified that $d_G(p_i^j, p_{i'}^j)$ satisfies the following fundamental conditions for distance metric functions [4]: (1)

1) *Non-negativity*: $d_G(\cdot, \cdot) \geq 0$
2) *Symmetry*: $d_G(p_i^j, p_{i'}^j) = d_G(p_{i'}^j, p_i^j)$
3) *Reflexivity*: $d_G(p_i^j, p_i^j) = 0$
4) *Identity of indiscernibles*: $d_G(p_i^j, p_{i'}^j) = 0 \iff \sum_{k=1}^l w_i^k \cdot p_i^{jk} = \sum_{k=1}^l w_{i'}^k \cdot p_{i'}^{jk}$
5) *Triangle inequality*: $d_G(p_i^j, p_{i''}^j) \leq d_G(p_i^j, p_{i'}^j) + d_G(p_{i'}^j, p_{i''}^j)$, for any $i \neq i' \neq i''$.

Example 1: Consider a MCGDM problem consisting of $n = 4$ alternatives, $m = 8$ experts and $l = 3$ evaluation criteria. Two experts e_i, e_2 provide the evaluation matrices $P_1 = (p_1^{jk})_{n \times l}$ and $P_2 = (p_2^{jk})_{n \times l}$ shown below, with $p_i^{jk} \in [0, 1]$.

$$P_1 = \begin{bmatrix} 0 & .75 & 1 \\ .25 & .25 & .5 \\ 1 & .5 & .75 \\ .75 & .5 & .5 \end{bmatrix} P_2 = \begin{bmatrix} .75 & 1 & 0 \\ .5 & .75 & 1 \\ 0 & 0 & .5 \\ .25 & .25 & .75 \end{bmatrix}$$

Assume that the importance weights vectors provided by e_1 and e_2 for the available criteria are, respectively, $W_1 = [.4\ .2\ .4]^T$ and $W_2 = [.2\ .4\ .4]^T$. Let us calculate the distance between e_1 and e_2 on the global performance of x_1. For e_1, we have that her (opinion on the) global performance of x_1 is $\mathcal{P}_1(x_1) = 0 \cdot 0.4 + 0.75 \cdot 0.2 + 1 \cdot 0.4 = 0.55$. On the other hand, for e_2 we have $\mathcal{P}_2(x_1) = 0.75 \cdot 0.2 + 1 \cdot 0.4 + 0 \cdot 0.4 = 0.55$. Based on this, $d_G(p_1^1, p_2^1) = |0.55 - 0.55| = 0$, i.e. the two experts are deemed as having an identical opinion on the *global* performance of x_1.

B. Distance based on Partial Alternative Performances

Let P_i, $P_{i'}$, p_i^j, $p_{i'}^j$, W_i and $W_{i'}$ be as introduced in Section III-A. We now introduce the distance between the opinions of e_i and $e_{i'}$ regarding the partial performances of x_j (i.e. how satisfactory x_j is deemed under each criterion). This distance function is denoted by $d_P(p_i^j, p_{i'}^j)$, and it is defined as:

$$d_P(p_i^j, p_{i'}^j) = \sum_{k=1}^{l} \left| p_i^{jk} - p_{i'}^{jk} \right|^{\sigma_{i,i'}^k} \tag{2}$$

with $\sigma_{i,i'}^k \in [0,1]$ a coefficient describing how similar the individual perspectives on the importance of $c_k \in C$ are among e_i and $e_{i'}$, i.e.,

$$\sigma_{i,i'}^k = 1 - |w_i^k - w_{i'}^k| \tag{3}$$

In Eq. (2), each addend $|p_i^{jk} - p_{i'}^{jk}|^{\sigma_{i,i'}^k} \in [0,1]$ represents the *weight-adjusted deviation* between (i) the partial performance of x_j under the kth criterion according to e_i, and (ii) the partial performance of x_j under the kth criterion according to $e_{i'}$. Figure 1 illustrates the behavior of $|p_i^{jk} - p_{i'}^{jk}|^{\sigma_{i,i'}^k}$. Consequently, $d_P(p_i^j, p_{i'}^j) \in [0,l]$, with $l \geq 2$ the number of evaluation criteria considered in the decision problem. Eq. (2) can thereby be normalized into the unit interval:

$$d_P(p_i^j, p_{i'}^j) = \frac{\sum_{k=1}^{l} \left| p_i^{jk} - p_{i'}^{jk} \right|^{\sigma_{i,i'}^k}}{l} \tag{4}$$

The weight similarity coefficient $\sigma_{i,i'}^k \in [0,1]$ plays the role of amplifying the deviation degree between assessments p_i^{jk} and $p_{i'}^{jk}$ when w_i^k and $w_{i'}^k$ are different from each other. For any *non weight-adjusted deviation* $|p_i^{jk} - p_{i'}^{jk}| \in [0,1]$, the lower $\sigma_{i,i'}^k$ (i.e. the more distinct weights w_i^k and $w_{i'}^k$ are), the closer its weight-adjusted counterpart $|p_i^{jk} - p_{i'}^{jk}|^{\sigma_{i,i'}^k}$ becomes to one. In other words, the deviation between an individual assessment p_i^{jk} and the associated $e_{i'}$'s assessment $p_{i'}^{jk}$ is quantified as higher if there also exists discrepancy among the importance both individuals assigned to the kth criterion.

It is likewise easy to verify that $d_P(p_i^j, p_{i'}^j)$ satisfies the following fundamental conditions inherent to a distance function: (1)

1) *Non-negativity*: $d_P(\cdot,\cdot) \geq 0$
2) *Symmetry*: $d_G(p_i^j, p_{i'}^j) = d_P(p_{i'}^j, p_i^j)$
3) *Reflexivity for any weighting vectors* $W_i, W_{i'}$: If $p_i^j = p_{i'}^j$, then $d_P(p_i^j, p_{i'}^j) = 0$, $\forall \sigma_{i,i'}^k \in [0,1]$.
4) *Identity of indiscernibles*: $d_P(p_i^j, p_{i'}^j) = 0 \iff p_i^{jk} = p_{i'}^{jk}$, $\forall k \in \{1, \ldots, l\}$.

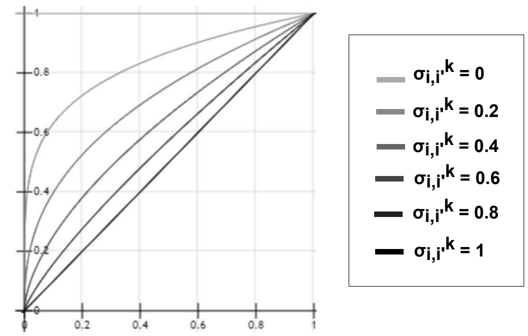

Fig. 1: Behavior of the weight-adjusted deviation $|p_i^{jk} - p_{i'}^{jk}|^{\sigma_{i,i'}^k}$ between p_i^{jk} and $p_{i'}^{jk}$, for different values of $\sigma_{i,i'}^k$

5) *Triangle inequality*: $d_P(p_i^j, p_{i''}^j) \leq d_P(p_i^j, p_{i'}^j) + d_P(p_{i'}^j, p_{i''}^j)$, for any $i \neq i' \neq i''$.

Some interesting particular cases of $d_P(p_i^j, p_{i'}^j)$ are described below.

- If $\sigma_{i,i'}^1 = \sigma_{i,i'}^2 = \ldots = \sigma_{i,i'}^l = 1$ then $d_P(p_i^j, p_{i'}^j) = (\sum_k |p_i^{jk} - p_{i'}^{jk}|)/l$, i.e. if the individual and collective criteria weighting vectors are identical, then their distance based on partial performances is purely based on the deviation between the assessments provided by $e_i, e_{i'}$ on x_j.
- Let e, $e_{i'}$ and $e_{i''}$ be three experts with identical assessments on x_j, i.e. $p_i^j = p_{i'}^j = p_{i''}^j$. Assume that $\forall k \in \{1, \ldots, l\}$ it holds $w_{i,i'}^k \geq w_{i,i''}^k$. Then $d_P(p_i^j, p_{i'}^j) \leq d(p_i^j, p_{i''}^j)$. Furthermore, if we also have that $\exists k \in \{1, \ldots, l\} : w_{i,i'}^k > w_{i,i''}^k$, then $d_P(p_i^j, p_{i'}^j) < d(p_i^j, p_{i''}^j)$.
- If $\sigma_{i,i'}^1 = \sigma_{i,i'}^2 = \ldots = \sigma_{i,i'}^l = 0$ then $d_P(p_i^j, p_{i'}^j) = 1$, i.e. if the weighting vectors from $e_i, e_{i'}$ are completely opposite to each other (which occurs when $w_i^k = 0$ and $w_{i'}^k = 1$ or vice versa for all criteria), then their distance based on partial performances is maximal, for any two *non-identical* assessment vectors p_i^j and $p_{i'}^j$.

Remark 1: The particular case when $\sigma_{i,i'}^k$ and $p_i^{jk} = p_{i'}^{jk}$ yields the indeterminate form 0^0 in the computation of the weight-adjusted deviation for c_j. Without loss of generality, we adopt the convention $0^0 = 0$, which implies a null weight-adjusted deviation when e_i and $e_{i'}$ assessments are identical.

Remark 2: The case when $\sigma_{i,i'}^k = 0$, $\forall k$ yields a drastic approach to measuring d_P, since $|p_i^{jk} - p_{i'}^{jk}|^{\sigma_{i,i'}^k} = 0$ if $p_i^{jk} = p_{i'}^{jk}$, and $|p_i^{jk} - p_{i'}^{jk}|^{\sigma_{i,i'}^k} = 1$ otherwise. In Section III-C we propose an adaptive approach for combining d_G and d_P into a consensus measure that softens this situation.

Example 2: Let P_1, P_2, W_1 amd W_2 be as shown in Example 1. Based on Eqs. (3) and (4), Table II summarizes the calculations of the non adjusted deviation, weight similarity coefficient and weight-adjusted deviation between e_1 and e_2 on x_1, for each criterion c_k, $k \in \{1, \ldots, l\}$. Consequently, $d_P(p_1^1, p_2^1) = (0.794 + 0.330 + 1)/3 = 0.708$, i.e. the two experts are deemed to have highly different opinions on the partial performances of x_1 (in contrast to their same opinion on the global performance of the alternative).

TABLE II: Pairwise deviation calculation for each criterion

c_k	$\left|p_i^{jk} - p_{i'}^{jk}\right|$	$\sigma_{i,i'}^k$	$\left|p_i^{jk} - p_{i'}^{jk}\right|^{\sigma_{i,i'}^k}$
c_1	0.75	0.8	0.794
c_2	0.25	0.8	0.330
c_3	1	1	1

C. Dual consensus measure based on global and partial alternative performances

Given the two distance functions d_G and d_P defined in the previous two subsections, we now combine them into a dual distance measure d_α, which in turn leads to defining a pairwise consensus measure at the alternative level, as follows:

$$CD_{i,i'}(x_j) = 1 - d_\alpha(p_i^j, p_{i'}^j) \quad (5)$$

where,

$$d_\alpha(p_i^j, p_{i'}^j) = \alpha \cdot d_G(p_i^j, p_{i'}^j) + (1-\alpha) \cdot d_P(p_i^j, p_{i'}^j) \quad (6)$$

with $\alpha \in [0,1]$ a parameter indicating the relative importance of the global performance distance function, with respect to the distance function based on partial performances. It was previously pointed out in Remark 2 that when two experts' perspectives on criteria are completely opposite, i.e. $\sigma_{i,i'}^k = 0\ \forall k$, the resulting pairwise distance d_P becomes 1. This implies that when all $\sigma_{i,i'}^k$ are close to zero, the resulting d_P tends to be too drastically high (even though there exist high similarities among both users at assessment level and in terms of a low d_G). To alleviate this situation and eliminate the need for manually setting α, below we propose a setting its value predicated on the similarity between experts' perspectives as:

$$\alpha = 0.5 + \frac{\sum_k |w_i^k - w_{i'}^k|}{2 \cdot l} \quad (7)$$

which restricts the parameter range to $\alpha \in [0.5, 1]$. When $|w_i^k - w_{i'}^k| = 1,\ \forall k$ then $\alpha = 1$, i.e. the consensus measure fully relies on the pairwise distance between global performance d_G exclusively. The opposite case occurs when $|w_i^k - w_{i'}^k| = 0,\ \forall k$, yielding $\alpha = 0.5$, hence both the global and partial distance views are equally taken into account within the consensus measure. We note that the above strategy in Eq. (7) should be interpreted as a particular example on flexibly determining α in the absence of domain expert knowledge, rather than being deemed as the only general solution to adopt in cojunction with the proposed dual consensus measure.

Finally, we formally describe how the proposed dual consensus measure can be instantiated as either of the two categories of consensus measures previously reviewed (see Section II-B).

- *Dual consensus measure based on distances to the collective preference*
  1) Calculate the collective decision matrix $P_c = (p_c^{jk})_{n \times l}$, with each assessment $p_c^{jk} = \phi(p_1^{jk}, p_2^{jk}, \ldots, p_m^{jk})$ and $\phi : [0,1]^m \to m$ an aggregation operator.
  2) For each $e_i \in E$ and $x_j \in X$, calculate $d_G(p_i^j, p_c^j)$.
  3) For each $e_i \in E$ and $x_j \in X$, calculate $d_P(p_i^j, p_c^j)$.
  4) For each $e_i \in E$ and $x_j \in X$, calculate $CD_{i,c}(x_j)$ based on Eqs. (5) and (6).
  5) For each $x_j \in X$, aggregate $CD_{1,c}(x_j), CD_{2,c}(x_j) \ldots CD_{m,c}(x_j)$ into $CD(x_j)$.
  6) Aggregate $CD(x_1) \ldots CD(x_n)$ into an overall consensus degree CD.
- *Dual consensus measure based on pairwise distances between experts*
  1) For each pair of experts $(e_i, e_{i'}) \in E \times E$, $i < i'$, and $x_j \in X$, calculate $d_G(p_i^j, p_{i'}^j)$.
  2) For each pair of experts $(e_i, e_{i'}) \in E \times E$, $i < i'$, calculate $d_P(p_i^j, p_{i'}^j)$.
  3) For each pair of experts $(e_i, e_{i'}) \in E \times E$, $i < i'$, calculate $CD_{i,i'}(x_j)$ based on Eqs. (5) and (6).
  4) For each $x_j \in X$, aggregate $CD_{1,2}(x_j), CD_{1,3}(x_j) \ldots CD_{m-1,m}(x_j)$ into $CD(x_j)$.
  5) Aggregate $CD(x_1) \ldots CD(x_n)$ into an overall consensus degree CD.

D. Application Example

We now illustrate the applicability of the dual consensus measure based on pairwise distances between experts in a logistics security scenario, as follows. Hazardous materials are substances capable of causing harm to humans, properties and the environment, such as explosives, flammables, oxidizing substances, poisonous gases, and radioactive materials. The release of hazardous materials caused by traffic accident or other potential risk factors during the transport can make a serious threat to residents, property and environment along their route. In order to reduce the risk of hazardous material transport, the enterprise should adopt a series of strategies to reduce the risk associated with the transport of hazardous material, among which the route selection plays an important role. The choice of the best suitable route is a complex decision-making problem, because the route with the minimal cost or travel time often pass through the area with high population or bad road condition. For a transport enterprise, there were four candidate routes available to transport the hazardous material, $n = 4$. Evaluating such alternatives in terms of three criteria ($l = 3$): efficiency, population density and road condition; the method proposed is applied to determine the most appropriate route. Six experts on secure logistics with diverse viewpoints on the importance of such three criteria, provide individual decision matrices with assessments expressed under the scale shown in Table I. The decision matrices and criteria weights elicited from e_1 and e_2 are as introduced in Example 2. Table III shows the assessments provided by the other four experts.

TABLE III: Assessments provided by e_3-e_6

e_i	e_3	e_4	e_5	e_6
p_i^1	(.5,1,.25)	(.5,.75,1)	(.25,.5,.25)	(1,.5,.5)
p_i^2	(.5,.5,.5)	(.5,0,.25)	(.5,.5,.5)	(1,.25,.25)
p_i^3	(0,0,.25)	(0,0,0)	(.5,.5,.25)	(0,0,.5)
p_i^4	(.75,.75,1)	(1,.75,.25)	(1,1,1)	(0,.25,.75)

Furthermore, $W_3 = [.1\ .8\ .1]$, $W_4 = [.5\ .2\ .3]$, $W_5 = [.4\ .3\ .4]$ and $W_6 = [.5\ .4\ .1]$. Without loss of generality, the arithmetic

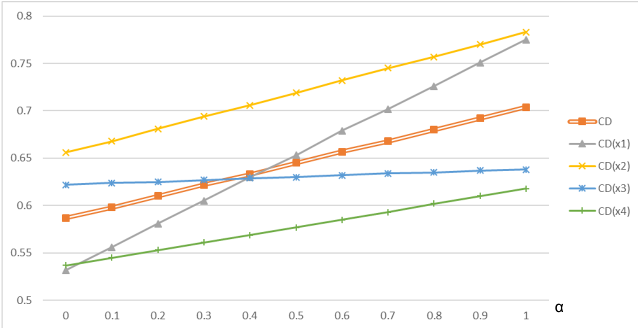

Fig. 2: Collective consensus degrees obtained at alternative and global level for different settings of parameter $\alpha \in [0,1]$.

mean operator is used to successively aggregate pairwise consensus degrees. The group consensus degrees are calculated for each alternative $x_j \in X$, when the control parameter α is dynamically calculated based on the criteria weights assigned by each pair of experts e_i, e_u, by using Eq. (7) (i.e. with $\alpha \in [0.5, 1]$), yielding: $CD(x_1) = .677$, $CD(x_2) = .734$, $CD(x_3) = .631$ and $CD(x_4) = .589$. By aggregating these consensus degrees, we obtain $CD = .657$.

To demonstrate the effect of α in the dual consensus measure, $CD(x_j)$, $j = 1, \ldots, 4$, and CD are now calculated for different values of α ranging across the complete unit interval [0,1]. The parameter value is fixed for the entire decision group (i.e. for the experiments conducted, the value of α is common to all pairs of experts instead of using Eq. (7)). The results are depicted in Figure 2. In view of the results obtained , it can be observed that: (i) increasing the relative importance of d_G with respect to d_P (by increasing α) yields a higher consensus degree both at global level and for all four alternatives considered, i.e. the dual consensus measure behaves more optimistically (resp. pessimistically) by increasing (resp. decreasing) α, hence considering the pairwise distances between partial performances of alternatives contributes to a stronger sensitivity towards disagreement positions between experts opinions; and (ii) the highest (resp. lowest) variability in the consensus degrees as α increases are observed for x_1 (resp. x_3). By analyzing the partial and dual distances between all the $m(m-1)/2 = 15$ different pairs of experts on each alternative, $d_P(p_i^j, p_{i'}^j)$ and $d_\alpha(p_i^j, p_{i'}^j)$, it was observed that x_1 presented the smallest differences between highest and lowest pairwise values for $d_P(p_i^j, p_{i'}^j), d_\alpha(p_i^j, p_{i'}^j)$, whereas x_3 presented the largest such differences. Importantly, the findings expounded above are specific to the example presented in this paper, hence it may not be generalized to any instance of multi-perspective MCGDM problem. A comprehensive experimental study on the behavior of the proposed dual consensus measure and its implications in diverse practical applications, constitutes our most immediate direction for future research.

IV. Concluding Remarks

In many real-life Multi-Criteria Group Decision Making (MCGDM) situations, participants may have diverse perspectives on the relative importance of evaluation criteria when providing their individual opinions on the existing alternatives. This paper introduced a novel dual consensus measure for reliably quantifying the collective agreement level among participants' opinions in such MCGDM problems, taking into consideration both the global performance of alternatives according to each individual opinions, and the partial performances of the alternatives under each criterion, along with the similarity between the perspectives of participants regarding the relative importance of criteria. Future work will focus on generalizing the proposed dual consensus measure to different multi-perspective MCGDM frameworks under diverse preference formats and the participation of mixed groups involving human experts and autonomous systems (e.g. in multi-agent planning, surveillance, intelligent transportation, and smart cities domains). We also aim at defining a complete consensus model for multi-perspective MCGDM incorporating the proposed dual consensus measure, and to extending its applicability to decision problems involving large, highly heterogeneous groups of participants exhibiting different behaviors during the process of building consensus.

Acknowledgments

All underlying data are included in full within this paper.

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